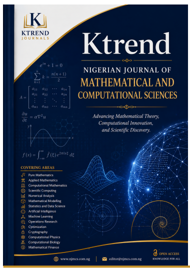

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Empirical Estimates of Stochastic Systems to Measure the Wealth of Corporate Investors

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Abstract. *Financial-market dynamics are inherently uncertain, and stochastic differential equations provide a natural framework for assessing the evolution of investment wealth under such conditions. This study formulates two stochastic systems for measuring the wealth of corporate investors by incorporating expected stock returns, intrinsic growth rates, interest-rate parameters, volatility, periodic effects, and random market fluctuations. The systems are solved analytically through Itô's lemma after logarithmic transformation of the wealth processes, yielding explicit expressions for the fourth and fifth corporate investors. Numerical evaluations are then used to examine the effects of intrinsic growth, interest rates, and volatility on portfolio values. The results show that increases in intrinsic growth rates generally increase investor wealth, whereas higher interest-rate parameters reduce wealth. Increased volatility lowers wealth under the non-periodic specification and makes wealth more sensitive to market fluctuations when periodic effects are incorporated. Surface-view representations further illustrate the response of investor wealth to changes in the principal model parameters. The findings provide a quantitative basis for corporate investment decisions under time-varying and uncertain market conditions and suggest that stochastic delay and periodic extensions may offer useful directions for subsequent research.*

Keywords: wealth; stochastic systems; financial mathematics; periodic events; stocks.

1. Introduction

The stock exchange market, also referred to as the share or equity market, is an important component of the capital market through which firms and other issuers can access long-term funding by issuing securities. Investors participate in these markets through the purchase and sale of financial instruments and may receive returns in the form of capital gains and dividends. By transferring funds from investors to equity issuers, stock markets play an important role in business development and economic growth.

The evolution of risky asset prices is commonly represented by diffusion processes defined on appropriate probability spaces. Geometric Brownian motion remains one of the standard reference models for stock-price dynamics [2]. A number of authors have investigated stochastic models for stock-market prices and investment values. Azor, Nwobi, and Amadi [3] studied systems of stochastic differential equations (SDEs) for economic investments whose rates of return and asset valuations involve price-index, periodic, and multiplicative effects. Amadi, Tamunotonye, and Azor [4] investigated asset-value models with delay parameters and obtained conditions governing the resulting asset values. Adeosun, Edeki, and Ugbebor [1] examined stochastic stock-price models and reported that their formulation was useful for predicting stock-market prices.

Ofomata et al. [5] considered selected stocks in the Nigerian Stock Exchange and estimated the drift and volatility coefficients of the corresponding stochastic differential equations. Dmouj [6] discussed geometric Brownian motion and its use in stock-price modelling, while Straja [7] examined the stochastic modelling of stock-price time series. Osu [2] studied stochastic variations in capital-market prices, and related contributions on stochastic stock-price behaviour, market equilibrium, and forecasting include [8-14] and [15-22].

Ekakaa, Nwobi, and Amadi [15] considered systems of interacting investors based on a Lotka–Volterra competition structure. The present study differs by incorporating stochastic parameters into the wealth equations and by examining intrinsic growth rates, interest rates, periodic effects, and volatility in the assessment of independent corporate-investor wealth. The specific objective is to obtain empirical estimates for two stochastic wealth systems representing the fourth and fifth corporate investors and to evaluate how the key parameters influence their portfolio values.

2. Materials and Methods

2.1. Mathematical Framework

Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$ be a filtered probability space and let

$$W(t) = (W_1(t), W_2(t), \ldots, W_m(t))^\top, \qquad t \geq 0,$$

be an m-dimensional Brownian motion. A general stochastic differential equation may be expressed as

$$dX(t) = f(t, X(t))\, dt + g(t, X(t))\, dW(t), \qquad 0 \leq t \leq T, \tag{2.1}$$

with initial condition

$$X(0) = X_0. \tag{2.2}$$

Equivalently,

$$X(t) = X_0 + \int_0^t f(s, X(s))\, ds + \int_0^t g(s, X(s))\, dW(s). \tag{2.3}$$

Theorem 2.1 (Existence and uniqueness)**.** *Let $T > 0$ and suppose that the coefficient functions*

$$f : [0,T] \times \mathbb{R}^n \to \mathbb{R}^n, \qquad g : [0,T] \times \mathbb{R}^n \to \mathbb{R}^{n\times m}$$

are continuous. Assume that there exist finite constants $\lambda, \beta > 0$ such that, for all $t \in [0,T]$ and $x, y \in \mathbb{R}^n$,

$$\|f(t,x) - f(t,y)\| + \|g(t,x) - g(t,y)\| \leq \lambda\|x - y\|, \tag{2.4}$$

$$\|f(t,x)\| + \|g(t,x)\| \leq \beta(1 + \|x\|). \tag{2.5}$$

If X_0 is an $\mathbb{R}^n$-valued random variable with $\mathbb{E}\|X_0\|^2 < \infty$, then the SDE admits a unique solution on $[0,T]$ satisfying the usual square-integrability condition [16,17].

Theorem 2.2 (Itô's lemma)**.** *Let $F(S,t)$ be twice continuously differentiable in S and once continuously differentiable in t, and suppose*

$$dS_t = a_t\, dt + b_t\, dW_t. \tag{2.6}$$

Then

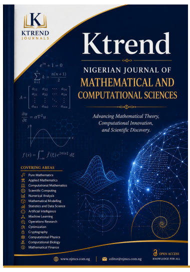
$$dF = \left(a_t \frac{\partial F}{\partial S} + \frac{\partial F}{\partial t} + \frac{1}{2} b_t^2 \frac{\partial^2 F}{\partial S^2} \right) dt + b_t \frac{\partial F}{\partial S}\, dW_t. \tag{2.7}$$

For a geometric Brownian motion

$$dS = \mu S\, dt + \sigma S\, dW_t, \tag{2.8}$$

Itô's lemma gives

$$dF = \left(\mu S \frac{\partial F}{\partial S} + \frac{\partial F}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 F}{\partial S^2} \right) dt + \sigma S \frac{\partial F}{\partial S}\, dW_t. \tag{2.9}$$



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2.2. Problem Formulation

Let the wealth of the fourth and fifth corporate investors be represented by $V_4(t)$ and $V_5(t)$, respectively. The stochastic systems are

$$dV_4(t) = (\mu\alpha_4 - \beta_4)V_4(t)\,dt + \sigma V_4(t)\,dW_t^4, \tag{2.10}$$
$$dV_5(t) = (K\tanh(\alpha_5) - \beta_5)V_5(t)\,dt + \sigma V_5(t)\,dW_t^5, \tag{2.11}$$

subject to

$$V_4(0) = V_{40} > 0, \qquad V_5(0) = V_{50} > 0. \tag{2.12}$$

Here μ is the expected rate of return on stock, σ is stock volatility, W_t^4 and W_t^5 are Wiener processes, K is a constant associated with the periodic return component, α_4 and α_5 are intrinsic growth-rate parameters, and β_4 and β_5 are interest-rate parameters.

2.3. Method of Solution

For the fourth investor, define

$$f(V_4(t), t) = \ln V_4(t).$$

Then

$$\frac{\partial f}{\partial V_4} = \frac{1}{V_4}, \qquad \frac{\partial^2 f}{\partial V_4^2} = -\frac{1}{V_4^2}, \qquad \frac{\partial f}{\partial t} = 0. \tag{2.13}$$

Applying Itô's lemma to (2.10) gives

$$d\ln V_4(t) = \left(\mu\alpha_4 - \beta_4 - \frac{1}{2}\sigma^2\right)dt + \sigma\,dW_t^4. \tag{2.14}$$

Integrating from 0 to t yields

$$\ln\frac{V_4(t)}{V_{40}} = \left(\mu\alpha_4 - \beta_4 - \frac{1}{2}\sigma^2\right)t + \sigma W_t^4, \tag{2.15}$$

and hence

$$V_4(t) = V_{40}\exp\left[\left(\mu\alpha_4 - \beta_4 - \frac{1}{2}\sigma^2\right)t + \sigma W_t^4\right]. \tag{2.16}$$

For the fifth investor, let $f(V_5(t), t) = \ln V_5(t)$. Applying the same procedure to (2.11) gives

$$d\ln V_5(t) = \left(K\tanh(\alpha_5) - \beta_5 - \frac{1}{2}\sigma^2\right)dt + \sigma\,dW_t^5. \tag{2.17}$$

Therefore,

$$V_5(t) = V_{50}\exp\left[\left(K\tanh(\alpha_5) - \beta_5 - \frac{1}{2}\sigma^2\right)t + \sigma W_t^5\right]. \tag{2.18}$$

Equations (2.16) and (2.18) are the stochastic wealth estimates used in the numerical analysis.

3. Results and Discussion

The simulation study uses the reported parameter values $\alpha_4 = 0.0260$, $\alpha_5 = 0.0360$, $\beta_4 = 0.0006$, $\beta_5 = 0.0007$, $\mu = 0.25$, $K = 1.0000$, $h = 25$, $\sigma = 0.03$, $t = 5$, $W_t^4 = W_t^5 = 1$, $V_{40} = 34.43$, and $V_{50} = 76.35$. The numerical values reported in the source manuscript are reproduced below.

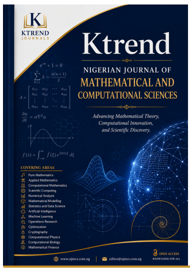

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Table 1: Effect of intrinsic growth rate on the wealth of the fourth corporate investor; $\mu = 0.25$, $\beta_4 = 0.0006$, $\alpha_4 = 0.0260$.

V_{40}	α_4	σ	t	$V_4(t)$	α_4	$V_4(t)$
60.77	0.1	0.20000	5.0000	75.8757	1.1	264.8322
	0.2	0.20000	5.0000	85.9784	1.2	300.09415
	0.3	0.20000	5.0000	97.4263	1.3	340.0512
	0.4	0.20000	5.0000	110.3985	1.4	385.3285
50.25	0.1	0.20000	5.0000	62.7407	1.1	218.9866
	0.2	0.20000	5.0000	71.0945	1.2	248.1443
	0.3	0.20000	5.0000	80.5607	1.3	281.1844
	0.4	0.20000	5.0000	91.2872	1.4	318.6336
40.10	0.1	0.20000	5.0000	50.0677	1.1	174.7535
	0.2	0.20000	5.0000	56.7342	1.2	198.02164
	0.3	0.20000	5.0000	64.2882	1.3	224.3879
	0.4	0.20000	5.0000	72.8481	1.4	254.2648
36.0000	0.1	0.20000	5.0000	44.9486	1.1	156.8859
	0.2	0.20000	5.0000	50.9334	1.2	177.7750
	0.3	0.20000	5.0000	57.7151	1.3	201.4455
	0.4	0.20000	5.0000	65.3998	1.4	228.2677

Table 2: Effect of interest rate on the wealth of the fourth corporate investor; $\mu = 0.25$, $\alpha_4 = 0.0260$, $\beta_4 = 0.0006$.

V_{40}	β_4	α_4	t	$V_4(t)$	β_4	$V_4(t)$
60.77	0.1	0.0260	5.0000	42.08010	0.8	1.2707
	0.2	0.0260	5.0000	25.5234	0.6	3.4542
	0.3	0.0260	5.0000	15.4807	0.4	9.3895
	0.4	0.0260	5.0000	9.3895	0.2	25.5234
50.25	0.1	0.0260	5.0000	34.7963	0.8	1.0508
	0.2	0.0260	5.0000	21.1050	0.6	2.8563
	0.3	0.0260	5.0000	12.8008	0.4	7.7641
	0.4	0.0260	5.0000	7.7641	0.2	21.1050
40.10	0.1	0.0260	5.0000	27.7677	0.8	0.8385
	0.2	0.0260	5.0000	16.8420	0.6	2.2793
	0.3	0.0260	5.0000	10.2152	0.4	6.1958
	0.4	0.0260	5.0000	6.1958	0.2	16.8420
36.0000	0.1	0.0260	5.0000	51.9883	0.8	0.7528
	0.2	0.0260	5.0000	15.12000	0.6	2.0463
	0.3	0.0260	5.0000	9.1707	0.4	5.5623
	0.4	0.0260	5.0000	5.5623	0.2	15.12000

Table 3: Effect of interest rate on the wealth of the fifth corporate investor; $\mu = 0.25$, $\alpha_5 = 0.0360$, $\beta_5 = 0.0007$.

V_{50}	β_5	α_5	t	$V_5(t)$	β_5	$V_5(t)$
60.77	0.1	0.0360	5.0000	41.5998	0.8	1.2562
	0.2	0.0360	5.0000	25.2316	0.6	3.4147
	0.3	0.0360	5.0000	15.3037	0.4	9.2822
	0.4	0.0360	5.0000	9.2822	0.2	25.2316
50.25	0.1	0.0360	5.0000	34.3984	0.8	1.0387
	0.2	0.0360	5.0000	20.8637	0.6	2.8236
	0.3	0.0360	5.0000	12.6545	0.4	7.6753
	0.4	0.0360	5.0000	7.6753	0.2	20.8637
40.10	0.1	0.0360	5.0000	27.4503	0.8	0.8289
	0.2	0.0360	5.0000	16.6494	0.6	2.2533
	0.3	0.0360	5.0000	10.09839	0.4	6.1250
	0.4	0.0360	5.0000	6.1250	0.2	16.6494
36.0000	0.1	0.0360	5.0000	24.6436	0.8	0.7442
	0.2	0.0360	5.0000	14.9471	0.6	2.0229
	0.3	0.0360	5.0000	9.0659	0.4	5.4987
	0.4	0.0360	5.0000	5.4987	0.2	14.9471

Tables 1–3 indicate that the wealth values respond strongly to both intrinsic growth and interest-rate parameters. In general, larger intrinsic growth rates produce larger wealth values. By contrast, increases in the interest-rate parameter reduce the present value of future cash flows and therefore reduce investor wealth. Lower interest rates have the opposite effect because future cash flows are discounted less heavily.

Table 4: Effect of stock volatility on the wealth of the fifth corporate investor; $\mu = 0.25$, $\beta_5 = 0.0007$, $\alpha_5 = 0.0360$, $K = 1.0000$, $h = 25$.

V_{50}	α_5	σ	t	$V_5(t)$	σ	$V_5(t)$
60.77	0.0360	0.50000	5.0000	54.5761	1.1	5.6103
	0.0360	0.60000	5.0000	45.8143	1.2	7.5731
	0.0360	0.70000	5.0000	21.6407	1.3	5.5323
	0.0360	0.80000	5.0000	12.4858	1.4	0.3087
50.25	0.0360	0.50000	5.0000	45.1283	1.1	4.6391
	0.0360	0.60000	5.0000	37.8833	1.2	6.2621
	0.0360	0.70000	5.0000	17.8944	1.3	4.5746
	0.0360	0.80000	5.0000	10.3244	1.4	0.2553
40.10	0.0360	0.50000	5.0000	36.01287	1.1	3.7020
	0.0360	0.60000	5.0000	30.2313	1.2	4.9972
	0.0360	0.70000	5.0000	14.2799	1.3	3.6505
	0.0360	0.80000	5.0000	8.2390	1.4	0.2037
36.0000	0.0360	0.50000	5.0000	32.3308	1.1	3.3235
	0.0360	0.60000	5.0000	27.1403	1.2	4.4863
	0.0360	0.70000	5.0000	12.8199	1.3	3.2773
	0.0360	0.80000	5.0000	7.3967	1.4	0.1829

Table 4 shows that higher volatility generally decreases the value of wealth because volatility increases uncertainty in the investment process. The periodic component in the fifth-investor model can also make terminal wealth more sensitive to changes in the volatility of the underlying assets. Consequently, the response to volatility need not be monotonic across all parameter combinations when periodic effects are present.

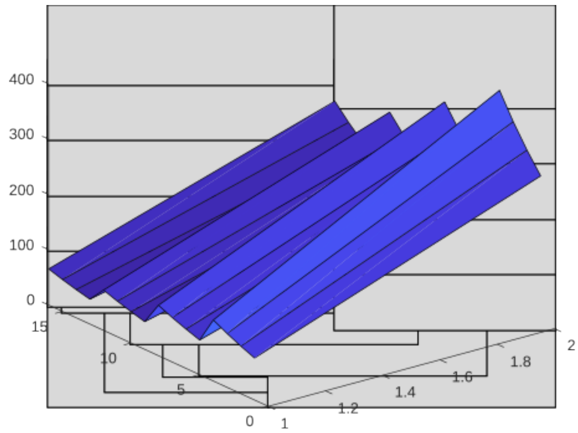

Figure 1: Surface view of the effect of intrinsic growth rate on the wealth of the fourth corporate investor.

The surface plot in Figure 1 illustrates the increasing trend in wealth as the intrinsic growth-rate parameter rises. The general monotonic pattern indicates that stronger intrinsic growth is associated with improved wealth accumulation under the stated parameter settings.

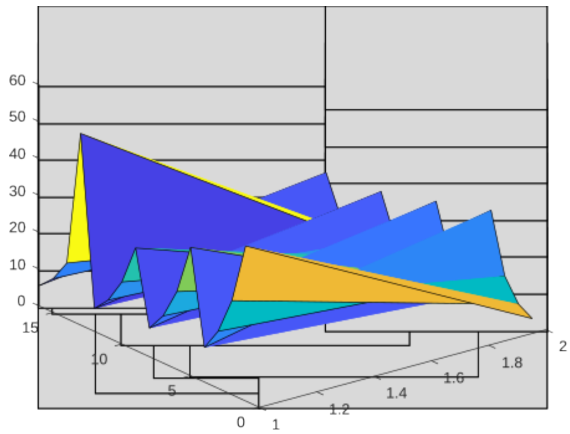

Figure 2: Surface view of the effect of interest rate on the wealth of the fourth corporate investor.

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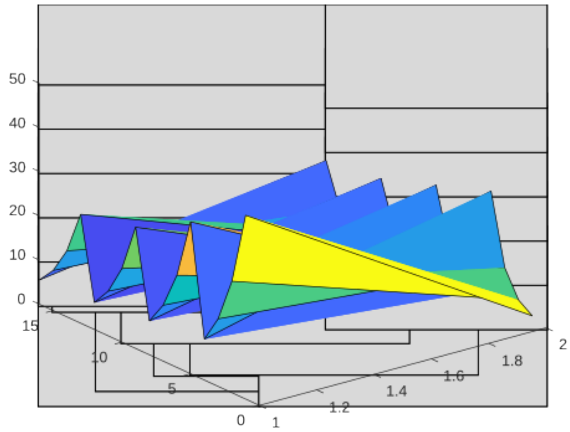

Figure 3: Surface view of the effect of interest rate on the wealth of the fifth corporate investor.

Figures 2 and 3 show the responses of the two corporate investors to changes in interest rates. The up-and-down movement reflects the stochastic nature of the wealth processes and the sensitivity of portfolio values to the selected parameter combinations.

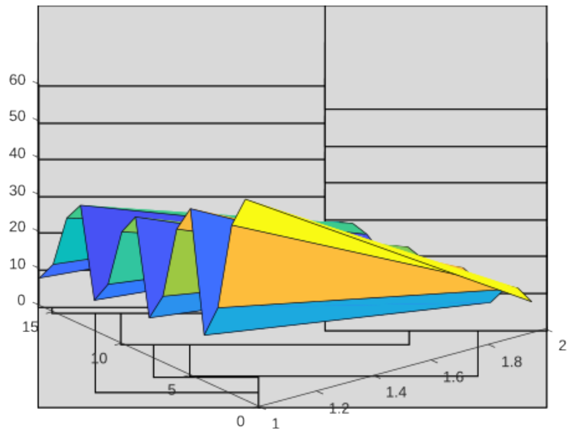

Figure 4: Surface view of the effect of stock volatility on the wealth of the fifth corporate investor.

Figure 4 represents the volatility response of the fifth corporate investor under a periodic investment specification. The observed fluctuations are consistent with time-varying market conditions and the stochastic structure of the model.

4. Conclusion

This study develops two stochastic differential-equation models for assessing the wealth of corporate investors and obtains analytical solutions using Itô's lemma. The results indicate that higher intrinsic growth rates tend to increase investor wealth, while higher interest-rate parameters reduce wealth. Volatility introduces additional uncertainty and, under the periodic specification, can make the wealth process more sensitive to changes in the underlying market environment. The surface-view representations provide additional visual evidence of these parameter effects and may assist investors in understanding the consequences of changing market conditions. Future studies may extend the framework by incorporating stochastic delay differential equations, correlated sources of uncertainty, time-varying coefficients, and empirical calibration using observed market data.

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