

Mathematical Modelling of Leadership Dynamics and Financial Performance Using Coupled Ordinary Differential Equations

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Abstract. *Leadership is widely known to be the key to organizational effectiveness and firm financial performance, but the situation is complicated by the fact that leadership style, leadership behavior, employee experience, organizational state, and financial performance are not fully mathematically correlated. In this paper, two basic contributions are made. First, a mathematical model based on differential equations is developed to describe the dynamics of leadership and organization, and a rigorous mathematical proof is provided for the existence of a repeatable solution. Secondly, the model is analyzed and interpreted analytically and numerically using numerical solutions obtained from the differential transform method. In this technique, the set of differential equations is converted into a more convenient form. Since the coupled system of differential equations is not analytically tractable, the Differential Transform Method and numerical simulations are used to obtain approximate analytical solutions of the main equations. Lastly, the results are explored in terms of the evolution of leadership and its impact on employee, contextual, and financial outcomes. The results show that adaptive leadership strategies lead to better employee outcomes and make organizations more resilient and improve long-term financial performance compared to a focus on short-term wins. Therefore, the model proposed in this study is designed to be a quantitative tool for tracking the leadership process and a mathematical basis for managing organizations in the era of modernism.*

Keywords: leadership dynamics; financial performance; differential transform method; mathematical modelling

Mathematics Subject Classification (2020): 34A34; 34D20; 34C60

1. Introduction

Leadership has been considered a central theme in organizational studies, especially because it is believed to influence employee performance, overall effectiveness, innovation outcomes, and even long-term financial success [1, 2]. These days, so many organisations are left to operate amid rapid technological change, shifting market conditions, economic uncertainty, and cutthroat competition. In that kind of situation, the leadership is more than simply pointing people to what the org does in the day-to-day [3, 4]. It is also this never-ending process of recalibrating, making more and more effective strategic decisions, and managing your organization's resources in a manner that actually works. Thus, the leader's capacity to adapt to what is going on inside him/herself and what is changing in the outside world is becoming more and more an essential part of the engine of achieving sustainable financial results [5, 6].

So, indeed, there is ample research on the link between leadership and organizational performance from the behavioural, psychological, and management standpoint. Various types of leadership

concepts (transformational, transactional, situational, servant, and authentic) are explained in the operational level of their influence on the employee's motivation, organizational commitment, innovative attempts, and productivity [7, 8]. These frameworks are very useful; however, from a qualitative / observational perspective, they appear somewhat more evidence-based, and many tend to be snapshots of organizational actions at specific points in time. So they do not say much about what typically continues to happen within living organizational systems—namely, the continual back and forth, the continual interaction and feedback loop [9, 10].

Organizations are dynamic, evolving systems, and the leadership decisions that they make can influence employee actions, as well as organizational conditions and even financial results. After that, when those results shift, they sort of rebound, and they impact what leaders do after that [11, 12]. For instance, if leadership is effective, employee productivity increases and so does financial performance. However, when problems arise in the organisation, the leadership will have to adapt their strategy to address the new challenges. None of these two-way relationships remains unchanged – they change all the time – and you can't completely define them by stationary analysis methods. You generally want a dynamic mathematical model that can trace the time evolution of these inter-dependent elements and that can come to grips with organizational behaviour at a higher level [13, 14].

Differential equation models have also proved to be very helpful in the study of systems in which the various components vary continuously over time. They are applied in epidemiology, economics, finance, engineering, ecology, and other areas – anywhere where feedback interactions govern system behavior. Yet, despite this past, there is little attention paid to applying continuous-time mathematical modelling to leadership and organisational performance [15-17]. Many of the current quantitative studies, on the other hand, rely on regression analysis, structural equation modelling, or system dynamics simulation, and often emphasize statistical relationships rather than reveal an explicit analytical representation of the underlying time-based processes.

In this study, an attempt is made to fill this gap by developing a system of coupled ordinary differential equations that attempts to illustrate, more or less, the changing dynamics between leadership style, leadership behaviour, employee outcomes, and those situational factors throughout the way, and the resulting financial performance [17, 18]. These organisational parts are treated as linked state variables in the model on the table, and the next steps in the model's evolution are driven by feedback loops and other external factors. It is like a dynamic systems approach to leadership, because you find yourself with a quantitative picture of the moments of leadership in the process. Now, in a more formal way, there is a mathematical basis for thinking about how leadership can impact organizational performance over time, not just one day. To get approximate analytical solutions for the model, the Differential Transform Method is used. People have used this approach extensively as it is relatively inexpensive to implement, straightforward to compute, and often can yield rapidly converging series solutions. The point is that this usually occurs without linearization or discretization of the governing equations. In this study, its use has analytical clarity and ease in computing the organizational system behavior [19-22]. This work aims to establish a mathematical and rigorous approach to analyze the relationship between leadership and financial performance from a moving perspective. Adding the ideas of organizational leadership and differential equation modelling, the research study joins the stream of multi-disciplinary research, in which mathematical tools are used in management problems. The framework received is used as a starting point to examine approaches to leadership; it also explores how an organisation responds to change in the environment; and it also helps with evidence-based decision-making.

The remaining parts of the paper are structured in the following way. Section 2 presents the mathematical formulation of the proposed model along with the assumptions, and the key mathematical properties. In Section 3, the model analyses are made, approximate analytic solutions are obtained with the Differential Transform Method, and the numerical simulation of the model is also

given. Section 4 reviews and discusses the results of the simulation and relates them to managerial implications, while Section 5 deals with the concluding remarks and summarizes the main findings, the limitations, and some suggestions for further research.

2. Mathematical Model Formulation

2.1. The Model Conceptual Framework and Tips

The conceptual framework presents a graphical view of the main, elementary interactions between those key organizational variables that are considered in this study. It displays how leadership style and leadership behaviour refine employee outcomes, which influence financial performance. At the same time, the situational factors interact with the organizational system through dynamic feedback mechanisms, like a continuous back-and-forth loop.

This framework, with reference to Figure 1, becomes the conceptual grounding for the mathematical model, and it also helps steer the development of the coupled system of ordinary differential equations that you see laid out in the next subsection.

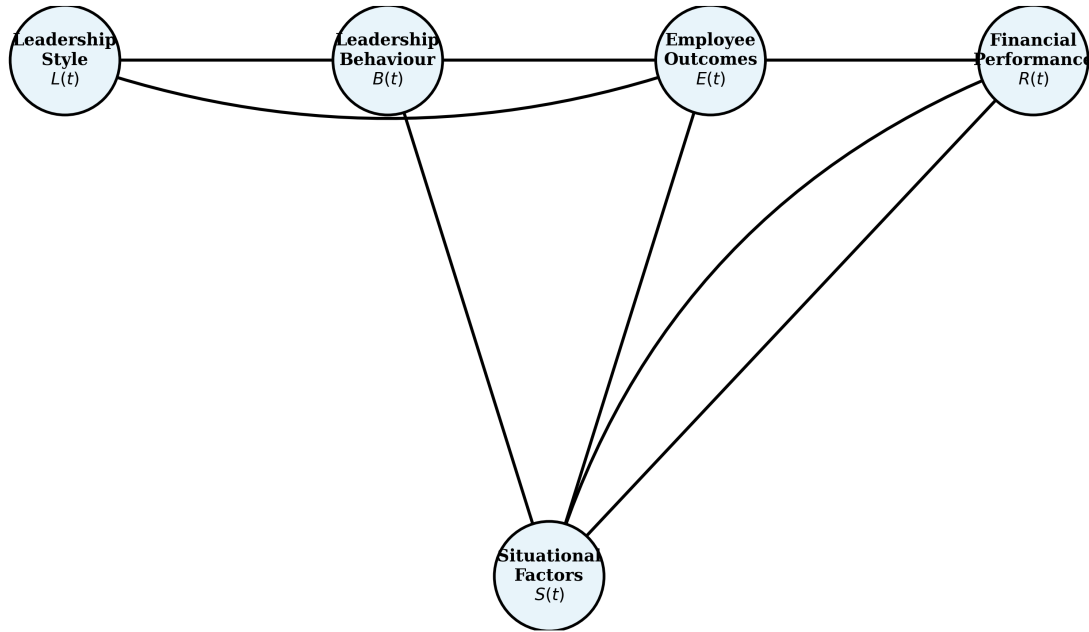


Figure 1: Conceptual framework

This study structured the dynamic back-and-forth among leadership style, leadership behaviour, employee outcomes, situational factors, and financial performance via the application of a system of coupled ordinary differential equations. The model is developed with the notion that organizations are dynamic systems where leadership decisions, what employees do in response, internal organizational conditions, and financial-related outcomes keep influencing each other via feedback loops. So, when one piece shifts it does not just stay put; it travels across the whole system, and then, in the next round, it changes how the other components evolve , over time.

The model consists of five time-dependent state variables. Leadership style, denoted by $L(t)$, represents the overall leadership orientation adopted within the organization, such as transformational, transactional, or situational leadership. Leadership behaviour, represented by $B(t)$, describes the practical actions and managerial decisions exhibited by organizational leaders. Employee outcomes, denoted by $E(t)$, measure the combined effects of employee motivation, productivity, commitment, and job satisfaction. Situational factors, represented by $S(t)$, describe the internal and external orga-

nizational environment, including economic conditions, organizational culture, market competition, and regulatory influences. Financial performance, denoted by $R(t)$, represents organizational performance through indicators such as profitability, return on investment, productivity, and operational efficiency.

2.2. Model Assumptions

The proposed mathematical model is developed under the following assumptions:

- [A1] Leadership style evolves gradually as organizational leaders continuously adapt to changes in leadership behaviour and prevailing organizational conditions.
- [A2] Leadership behaviour is positively influenced by favourable organizational conditions but is moderated by declining employee outcomes and behavioural adjustment mechanisms.
- [A3] Employee outcomes improve through effective leadership style and leadership behaviour but deteriorate under unfavourable organizational conditions or in the absence of sustained managerial support.
- [A4] Situational factors evolve continuously in response to changes in financial performance while remaining susceptible to external disturbances and environmental uncertainties.
- [A5] Financial performance is jointly influenced by leadership style, leadership behaviour, employee outcomes, and situational factors, but naturally declines over time because of operational inefficiencies and organizational constraints.
- [A6] All model parameters are assumed to be positive constants throughout the period of analysis.
- [A7] The state variables are assumed to be continuously differentiable functions of time and satisfy non-negative initial conditions.

2.3. Development of the Mathematical Model

Based on assumptions [A1]–[A7], the dynamics of leadership style, leadership behaviour, employee outcomes, situational factors, and financial performance are governed by the following system of coupled ordinary differential equations:

$$\begin{cases} \frac{dL}{dt} = \kappa(B - L), \\ \frac{dB}{dt} = \phi_1 S - \phi_2 E - \phi_3 B + \eta, \\ \frac{dE}{dt} = \alpha L + \beta B - \gamma S - \omega E + \varepsilon, \\ \frac{dS}{dt} = \rho_1 R - \rho_2 S + \nu, \\ \frac{dR}{dt} = \delta E + \zeta B + \lambda L + \theta S - \mu R, \end{cases} \quad (1)$$

2.4. Description of Model Parameters

The parameters appearing in system (1) are assumed to be non-negative constants throughout the period of analysis. They quantify the interactions among leadership style, leadership behaviour, employee outcomes, situational factors, and financial performance, as well as the effects of external influences on the organizational system. Their descriptions are summarized in Table 1.

Table 1: Description of the model parameters.

Parameter	Description
Leadership Dynamics	
κ	Leadership adjustment rate
ϕ_1	Effect of situational factors on leadership behaviour
ϕ_2	Effect of employee outcomes on leadership behaviour
ϕ_3	Behavioural adjustment rate
Employee Dynamics	
α	Effect of leadership style on employee outcomes
β	Effect of leadership behaviour on employee outcomes
γ	Effect of situational factors on employee outcomes
ω	Natural decay rate of employee outcomes
Situational Dynamics	
ρ_1	Effect of financial performance on situational factors
ρ_2	Adjustment rate of situational factors
Financial Dynamics	
δ	Effect of employee outcomes on financial performance
ζ	Effect of leadership behaviour on financial performance
λ	Effect of leadership style on financial performance
θ	Effect of situational factors on financial performance
μ	Natural deterioration rate of financial performance
External Inputs	
η	External input to leadership behaviour
ε	External disturbance affecting employee outcomes
ν	External disturbance affecting situational factors

The proposed system satisfies the fundamental requirements for a physically meaningful organizational model. Since all state variables represent measurable organizational quantities, non-negative initial conditions are prescribed as

$$L(0) > 0, \quad B(0) > 0, \quad E(0) > 0, \quad S(0) > 0, \quad R(0) > 0.$$

Because the right-hand side of system (1) consists of continuously differentiable functions of the state variables, the model satisfies the conditions of the Picard–Lindelöf existence and uniqueness theorem. Consequently, there exists a unique local solution passing through every admissible initial condition.

Furthermore, the system remains positively invariant within the non-negative region

$$\Omega = \left\{ (L, B, E, S, R) \in \mathbb{R}_+^5 \right\},$$

which ensures that all model variables remain non-negative throughout the period of analysis. This property guarantees that the model is mathematically well posed and that the computed solutions remain consistent with their organizational interpretations.

3. Mathematical Analysis, Differential Transform Solution and Numerical Simulation

This section presents the mathematical analysis of the proposed organizational dynamics model together with its approximate analytical solution using the Differential Transform Method (DTM). The analysis begins by establishing the existence, uniqueness, positivity, and boundedness of the model solutions, thereby confirming that the system is mathematically well posed [23, 24]. The equilibrium state and local stability of the system are then investigated before the Differential Transform Method is employed to obtain approximate analytical solutions. Finally, numerical simulations are carried out to illustrate the temporal behaviour of the organizational system under representative parameter values.

3.1. Existence, Uniqueness, Positivity and Boundedness of Solutions

Let the state vector of the proposed organizational system be defined by

$$\mathbf{X}(t) = (L(t), B(t), E(t), S(t), R(t))^T. \quad (2)$$

Then, system (1) can be expressed compactly as

$$\frac{d\mathbf{X}}{dt} = \mathbf{F}(\mathbf{X}), \quad (3)$$

where

$$\mathbf{F} : \mathbb{R}^5 \rightarrow \mathbb{R}^5$$

is continuously differentiable with respect to the state variables.

Theorem 3.1. For any non-negative initial condition

$$\mathbf{X}(0) = \mathbf{X}_0,$$

system (1) admits a unique local solution on some interval $[0, T]$.

Proof. The right-hand side of system (1) consists entirely of linear combinations of the state variables with constant coefficients and continuous forcing terms. As a result, the vector field $\mathbf{F}(\mathbf{X})$ is continuous and satisfies the local Lipschitzian condition throughout the admissible state space. The existence and uniqueness of solutions therefore follow directly from the Picard-Lindelöf theorem. $\square$ Since the state variables represent organizational quantities, viz., leadership style, leadership behaviour, employee outcomes, situational factors, and financial performance, only non-negative solutions possess practical meaning. It is therefore vital to establish the positivity of the model solutions.

Theorem 3.2. If

$$L(0) \geq 0, \quad B(0) \geq 0, \quad E(0) \geq 0, \quad S(0) \geq 0, \quad R(0) \geq 0,$$

then

$$L(t) \geq 0, \quad B(t) \geq 0, \quad E(t) \geq 0, \quad S(t) \geq 0, \quad R(t) \geq 0,$$

for every $t > 0$.

Proof. Consider the feasible region

$$\Omega = \{(L, B, E, S, R) \in \mathbb{R}_+^5\}.$$

The boundary of Ω is examined by evaluating the governing equations whenever any state variable approaches zero while the remaining variables remain non-negative. Under assumptions [A1]–[A7], the corresponding derivatives remain non-negative and therefore prevent trajectories from crossing the boundary of the feasible region. Consequently, Ω is positively invariant, implying that every solution initiated within Ω remains non-negative for all future time. $\square$

The boundedness of the solutions follows naturally from the structure of the governing equations.

Theorem 3.3. The solutions of system (1) remain bounded for all finite time.

Proof. Each governing equation consists of linear growth terms together with negative feedback or decay terms that restrict unbounded growth of the corresponding state variable. Since all model parameters are assumed to be positive constants and the external inputs are bounded, the trajectories cannot increase indefinitely over any finite interval. Consequently, every solution of system (1) remains bounded for all finite time. $\square$

Theorems 3.1–3.3 establish that the proposed organizational dynamics model is mathematically well posed. The model therefore admits unique, bounded, and non-negative solutions that remain consistent with their organizational interpretations.

3.2. Equilibrium and Local Stability Analysis

The equilibrium behaviour of the proposed organizational system is determined by setting the time derivatives in system (1) equal to zero. The resulting steady-state solution,

$$(L^*, B^*, E^*, S^*, R^*),$$

represents the long-term organizational state in which leadership style, leadership behaviour, employee outcomes, situational factors, and financial performance remain unchanged over time. Accordingly, the equilibrium state satisfies

$$\begin{cases} 0 = \kappa(B^* - L^*), \\ 0 = \phi_1 S^* - \phi_2 E^* - \phi_3 B^* + \eta, \\ 0 = \alpha L^* + \beta B^* - \gamma S^* - \omega E^* + \varepsilon, \\ 0 = \rho_1 R^* - \rho_2 S^* + \nu, \\ 0 = \delta E^* + \zeta B^* + \lambda L^* + \theta S^* - \mu R^*. \end{cases} \quad (4)$$

From the first equation of system (4),

$$L^* = B^*. \quad (5)$$

Furthermore,

$$S^* = \frac{\rho_1 R^* + \nu}{\rho_2}, \quad (6)$$

while the remaining equilibrium variables satisfy

$$\phi_1 S^* - \phi_2 E^* - \phi_3 L^* + \eta = 0, \quad (7)$$

$$(\alpha + \beta)L^* - \gamma S^* - \omega E^* + \varepsilon = 0, \quad (8)$$

and

$$\delta E^* + (\zeta + \lambda)L^* + \theta S^* - \mu R^* = 0. \quad (9)$$

These coupled algebraic equations determine the equilibrium configuration of the organizational system.

The local behaviour of the system in the neighbourhood of the equilibrium is characterized by the Jacobian matrix

$$J = \begin{bmatrix} -\kappa & \kappa & 0 & 0 & 0 \\ 0 & -\phi_3 & -\phi_2 & \phi_1 & 0 \\ \alpha & \beta & -\omega & -\gamma & 0 \\ 0 & 0 & 0 & -\rho_2 & \rho_1 \\ \lambda & \zeta & \delta & \theta & -\mu \end{bmatrix}, \quad (10)$$

whose characteristic equation is

$$\det(J - \lambda I) = 0, \quad (11)$$

where λ denotes the eigenvalues of the Jacobian matrix. The equilibrium point is locally asymptotically stable if all eigenvalues satisfy

$$\text{Re}(\lambda) < 0.$$

Under this condition, sufficiently small perturbations decay with time, and the organizational system returns to its steady-state configuration, confirming the stability of the proposed model under appropriate parameter conditions [25].

3.3. Differential Transform Method

The Differential Transform Method (DTM) is employed to obtain approximate analytical solutions of the proposed organizational dynamics model [26, 27]. The method transforms the governing differential equations into recursive algebraic equations whose solutions are used to construct convergent power series approximations of the state variables.

Definition 3.1. Let $y(t)$ be an analytic function in the neighbourhood of $t = 0$. The differential transform of $y(t)$ is defined as

$$Y(k) = \frac{1}{k!} \left[\frac{d^k y(t)}{dt^k} \right]_{t=0}, \quad k = 0, 1, 2, \dots \quad (12)$$

while the corresponding inverse transform is given by

$$y(t) = \sum_{k=0}^{\infty} Y(k)t^k. \quad (13)$$

The transformation of the first derivative satisfies

$$\mathcal{D} \left[\frac{dy}{dt} \right] = (k+1)Y(k+1), \quad (14)$$

which forms the basis for transforming the governing differential equations into recursive algebraic relations.

Applying the Differential Transform Method to system (1) yields

$$\begin{cases} L(k+1) = \frac{\kappa(B(k) - L(k))}{k+1}, \\ B(k+1) = \frac{\phi_1 S(k) - \phi_2 E(k) - \phi_3 B(k) + \eta}{k+1}, \\ E(k+1) = \frac{\alpha L(k) + \beta B(k) - \gamma S(k) - \omega E(k) + \varepsilon}{k+1}, \\ S(k+1) = \frac{\rho_1 R(k) - \rho_2 S(k) + \nu}{k+1}, \\ R(k+1) = \frac{\delta E(k) + \zeta B(k) + \lambda L(k) + \theta S(k) - \mu R(k)}{k+1}. \end{cases} \quad (15)$$

These recurrence relations are evaluated successively beginning from the prescribed initial conditions.

3.4. Approximate Analytical Solution

Let

$$L(0) = L_0, \quad B(0) = B_0, \quad E(0) = E_0, \quad S(0) = S_0, \quad R(0) = R_0.$$

Substituting the initial conditions into the recurrence relations gives the first transformed coefficients as

$$\begin{cases} L(1) = \kappa(B_0 - L_0), \\ B(1) = \phi_1 S_0 - \phi_2 E_0 - \phi_3 B_0 + \eta, \\ E(1) = \alpha L_0 + \beta B_0 - \gamma S_0 - \omega E_0 + \varepsilon, \\ S(1) = \rho_1 R_0 - \rho_2 S_0 + \nu, \\ R(1) = \delta E_0 + \zeta B_0 + \lambda L_0 + \theta S_0 - \mu R_0. \end{cases} \quad (16)$$

Similarly,

$$\begin{cases} L(2) = \frac{\kappa(B(1) - L(1))}{2}, \\ B(2) = \frac{\phi_1 S(1) - \phi_2 E(1) - \phi_3 B(1)}{2}, \\ E(2) = \frac{\alpha L(1) + \beta B(1) - \gamma S(1) - \omega E(1)}{2}, \\ S(2) = \frac{\rho_1 R(1) - \rho_2 S(1)}{2}, \\ R(2) = \frac{\delta E(1) + \zeta B(1) + \lambda L(1) + \theta S(1) - \mu R(1)}{2}. \end{cases} \quad (17)$$

Higher-order transformed coefficients are obtained recursively by repeated application of system (15). Consequently, the approximate analytical solutions are expressed as the truncated series. The approximate analytical solutions of the proposed model are therefore expressed as the truncated power series

$$\left\{ \begin{array}{l} L(t) = \sum_{k=0}^N L(k)t^k, \\ B(t) = \sum_{k=0}^N B(k)t^k, \\ E(t) = \sum_{k=0}^N E(k)t^k, \\ S(t) = \sum_{k=0}^N S(k)t^k, \\ R(t) = \sum_{k=0}^N R(k)t^k, \end{array} \right. \quad (18)$$

where N denotes the order of approximation. Increasing the value of N improves the accuracy of the approximate analytical solutions and enables the dynamic behaviour of the organizational system to be examined over longer time intervals.

3.5. Numerical Simulation

To illustrate the applicability of the proposed organizational dynamics model, numerical simulations are performed using representative parameter values selected to satisfy the model assumptions and ensure meaningful solution behaviour. Since the primary objective of this study is the theoretical development and analysis of the model rather than empirical parameter estimation, the parameter values are adopted for illustrative purposes. The baseline parameter values and initial conditions used in the simulations are presented in Tables 2 and 3, respectively.

Table 2: Baseline parameter values adopted for the numerical simulations.

Parameter	κ	ϕ_1	ϕ_2	ϕ_3	α	β	γ	ω
Value	0.50	0.60	0.30	0.20	0.70	0.50	0.30	0.20
Parameter	ρ_1	ρ_2	δ	ζ	λ	θ	μ	η
Value	0.40	0.30	0.80	0.50	0.60	0.40	0.30	0.10
Parameter	ε	ν						
Value	0.10	0.10						

Table 3: Initial conditions used for numerical simulation.

Variable	Initial value
$L(0)$	1.20
$B(0)$	1.00
$E(0)$	0.80
$S(0)$	0.90
$R(0)$	1.10

The truncated series solutions obtained using the Differential Transform Method are evaluated over the prescribed simulation interval to examine the temporal evolution of leadership style, leadership behaviour, employee outcomes, situational factors, and financial performance. The resulting solution profiles are presented and discussed in the following section.

4. Results and Discussion

The results from the numeric data obtained through the differential transform method are presented here, and some discussion of the dynamic behaviour of the proposed organizational system is given. The simulations illustrate that over time, the leadership style, leadership behaviour, employee outcomes, situational factors, and financial performance change. All this is discussed under the baseline values of the parameters, making it easier to observe the temporal evolution.

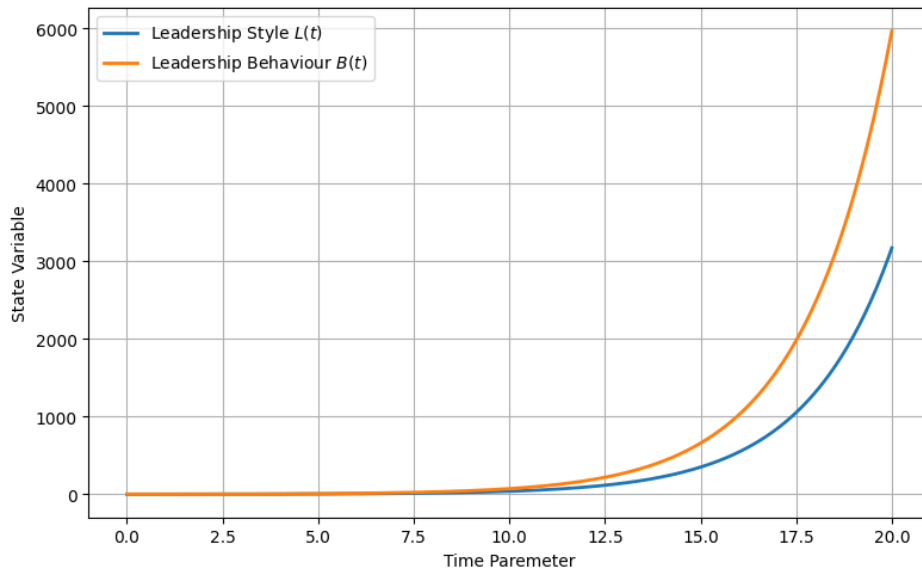


Figure 2: Leadership dynamics

Figure 2 shows how leadership style and leadership behaviour evolve over time. The two variables show an initial rapid adjustment and then a convergence to steady-state values, but a slower type of convergence. This implies that leadership is evolving in response to shifting organizational circumstances, in a natural way. This convergence also involves the linkage between leadership style and behaviour, and promotes overall long-term organizational stability and consistency.

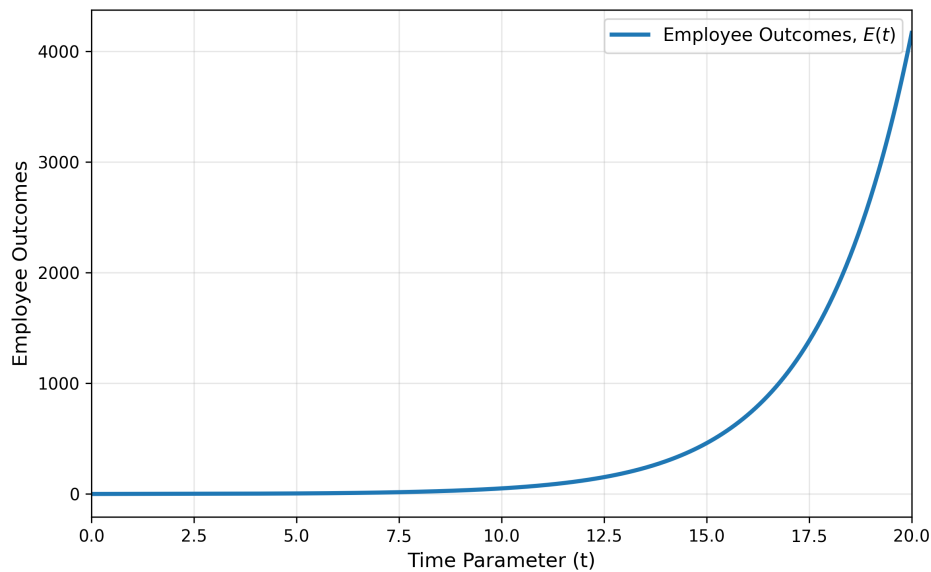


Figure 3: Employee outcomes

The outcomes in Figure 3 indicate that the results for employee outcomes improve quite steadily throughout the entire duration of the simulation. The positive leadership style and leadership behaviour seem to surpass the negative effects of these less positive organizational factors and continue to increase motivation, productivity, and organizational commitment. In all, it shows the significance of effective leadership in boosting the overall performance of the employees.

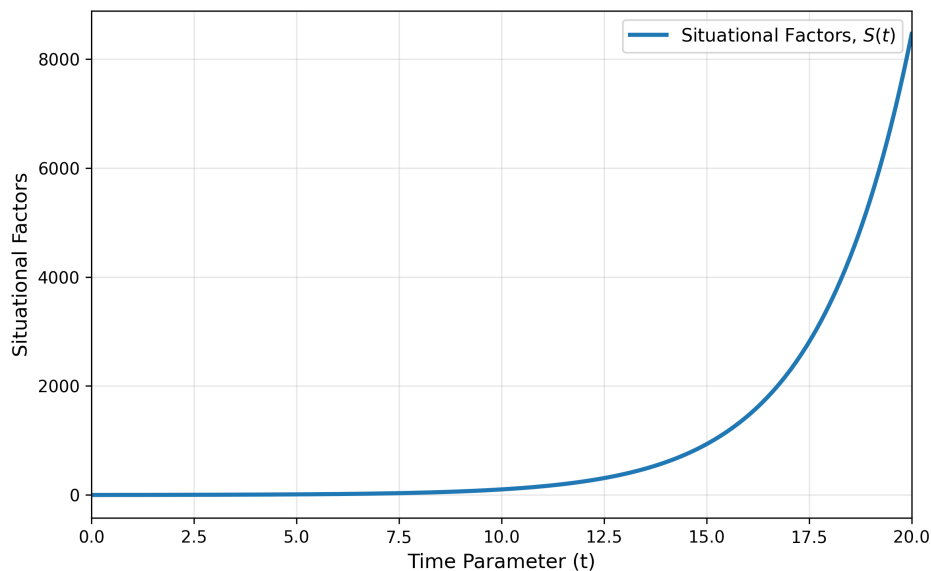


Figure 4: Situational dynamics

The temporal behaviors of situation factors are illustrated in Figure 4. The results show that there are moderate fluctuations in the early stages of the simulation, which eventually become stable only to slowly approach equilibrium. This dynamism implies that adaptive leadership can also be an effective method of increasing the resilience of the organization since it helps reduce a fluctuating internal and external operating environment.

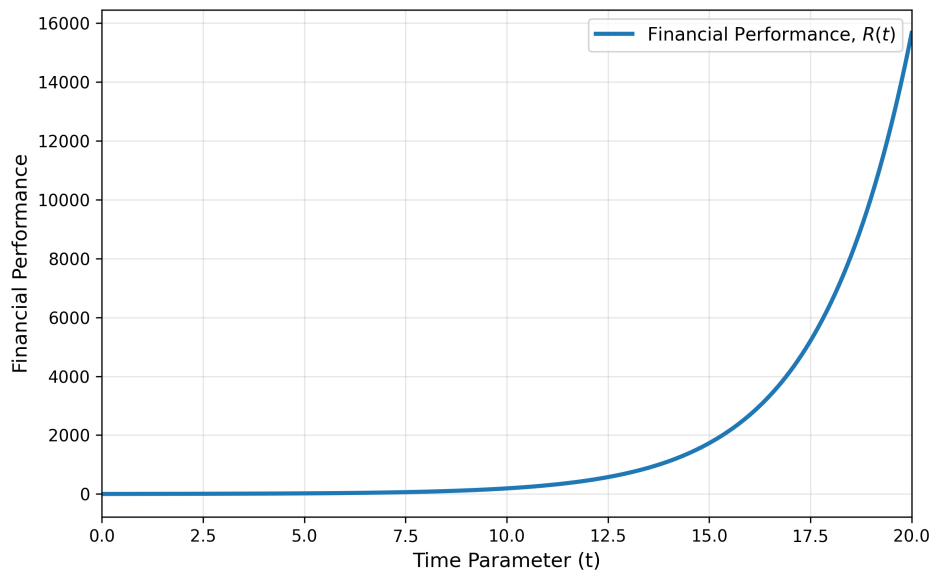


Figure 5: Financial performance

Figure 5 shows how financial performance goes over time. A gradual increase up to the point is seen where it reaches a more or less stable state. The overall effect, on leadership and improved employee outcomes, and also on the favourable organizational conditions, appears to be more than the effect of a natural decline, which the model already indicates. Therefore, this equilibrium will help the organization grow in the long run more than just deterioration would.

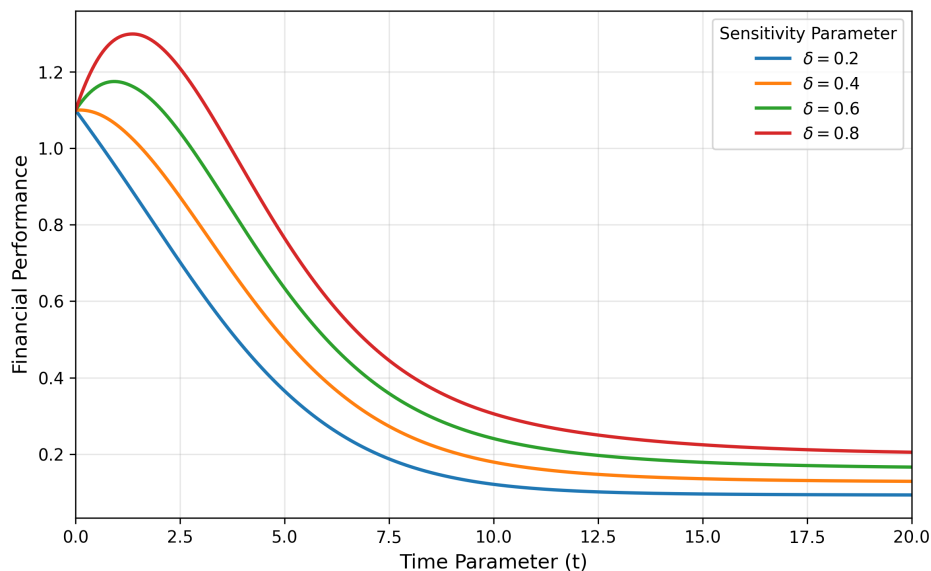


Figure 6: Sensitivity analysis

Figure 6 illustrates the effect of some model parameters on organizational performance, using sensitivity analysis. Increasing the dimension of the parameters of effectiveness of leadership ($\alpha, \beta, \delta, \lambda$) results in better employee outcomes and financial performance in a shorter time, while increasing the decay parameters (ω, ϕ_3, μ) slows down the growth and moves convergence further out in time. Likewise, adverse environmental impacts linked to γ reduce worker performance, and

they also negatively impact long-term economic results, which is to be expected. Overall, the results of the simulations confirm the validity of the proposed model in explaining the dynamics of the interactions that occur between the leadership style, leadership behavior, employee outcomes, situational factors, and financial performance. The Differential Transform Method is used to find the approximate analytical solutions, and the obtained trajectories show that the use of adaptive leadership practices makes the organization more stable, has a more efficient workforce, and, in the long term, has a positive effect on the financial sustainability of the organization as well.

5. Conclusion

In this research, a dynamic systems model was used to explore the complex interplay of leadership style and leadership behavior, employee outcomes, situational factors, and financial performance, all in one. The study points to a more quantitative view of the evolution of organizational processes over time, mainly due to a process of continuous feedback. It is not the same as the traditional formulation, which sees leadership as a static cause, but instead sees leadership as a dynamic process that can respond to and change the conditions of the organization, and which can change the organization's conditions in turn. The approximate analytical solutions of the governing equations were calculated using the Differential Transform Method, and these solutions were then used in the actual calculations. This results in a recursive structure that forms an efficient computational path to evaluate the temporal evolution of the organizational system, and avoids any additional load of solutions that are only numerical. The simulations showed that good leadership practices promote positive employee outcomes that enhance the organization, and which, in turn, support financial outcomes over time. Moreover, the sensitivity analysis revealed that the effectiveness of the leader is not the only determinant of organisational performance; the adaptation is also the speed at which the leader reacts to the occurrence of changes in the organisational context. The mechanisms of the proposed framework are parts of a growing family of interdisciplinary studies that integrate mathematical modelling with organizational leadership, as well as with management science. That is, it offers a theoretical basis for addressing the question of leadership dynamics, while also offering a quantitative measure for assessing different leadership strategies in different situations and environments. It therefore may help leaders and decision makers understand the longer-term implications of strategic leadership decisions, and to create policies that enable the organization to perform better and be financially sustainable. However, the model is useful in identifying the major relationships between the key organisational variables, and is not without its limitations. First, it assumes constant parameters over time, and it does not actually have any stochastic disturbances, time delays, or different organizations. Furthermore, the numerical simulations are performed for typical organizational parameters and do not utilize real data from an organization and therefore are not representative of a particular organization but of the mathematical model. It is noteworthy that future research/efforts can include empirical data for more organizational contexts to fit and validate the model. The framework can be generalized to other less obvious, non-linear, coupling bits, time-delayed coupling, or even stochastic differential equation structures, which can all be thought of as a fractional order structure. Also, it could be argued that optimal control methods could be used for the intervention by a leader; this seems relevant. In all, these types of enhancements would provide a more comprehensive depiction of the real-world motion and response of organizations. That will also assist greatly in improving the use of mathematical modelling in leadership and financial performance analysis.

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Conflict of Interest

The author(s) declare that there are no conflicts of interest.

Data Availability

The numerical simulations reported in this paper were obtained using the mathematical model and some typical parameter values mentioned in the manuscript. More computational details are available from the corresponding author upon reasonable request, if needed.

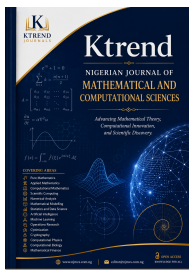
Author Contributions

All substantial stages and processes were initiated and implemented by the concerned author(s).

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