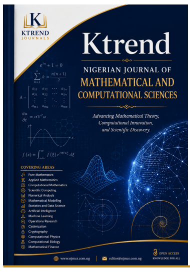

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Stress Testing Nigerian Banks: An Exponential-Decay Geometric Brownian Motion Model

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Abstract. *This paper applies an exponential-decay geometric Brownian motion (GBM) model to illustrate how shocks and sustained downward pressure may affect Nigerian bank stock prices. The standard GBM drift is adjusted from μ to $\mu - k$, where $k > 0$ is the decay rate. Brownian-motion paths illustrate the source of uncertainty, while simulated stock-price paths demonstrate how assets with identical initial values can diverge under volatility. Sensitivity analysis shows that the relation between k and μ determines the behavior of the expected price: the mean grows when $\mu > k$, remains constant when $\mu = k$, and declines exponentially when $\mu < k$. When $k > \mu$, the mean-price half-life is $\ln(2)/(k - \mu)$. The model therefore offers a transparent scenario-generation tool for regulators and risk managers. It is an illustrative stress-testing model rather than an empirically calibrated model of any named Nigerian bank.*

Keywords: stochastic processes; bank stress testing; geometric Brownian motion; exponential decay; stock prices

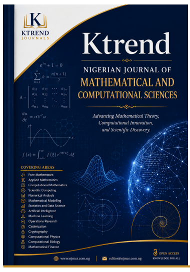
Mathematics Subject Classification (2020): 60H10; 60G15; 91G30; 91B70

1. Introduction

Stress-testing models should capture both random shocks and persistent changes in market conditions after extreme events. Nigerian bank stocks may respond sharply to monetary-policy announcements, foreign-exchange adjustments, or liquidity constraints. A conventional GBM treats the log-price shock as permanent and uses a constant drift. This paper introduces a parsimonious decay adjustment by replacing the drift μ with the net rate $\mu - k$, where $k > 0$ represents sustained decay pressure.

Stochastic models have been widely applied to asset prices, investment decisions, insurance risk, and market stability. Amadi et al. examined stochastic wealth dynamics under alternative return structures [1]. Wokoma et al. studied stock-price variation using a stochastic model [2], while Osu and Amadi analyzed fluctuations in two asset values [3]. Optimal investment and ruin-minimization problems were treated by Browne [4]; related reinsurance-investment problems appear in Bai and Guo [5], Liang and Bayraktar [6], and Ihedioha and Osu [7]. Extensions involving jump diffusion, stochastic volatility, and constant-elasticity-of-variance dynamics were investigated by Zhao et al. [8], Gu et al. [9], Lin and Li [10], and Liang et al. [18].

Within the stock-market literature, Osu and Okoroafor quantified random stock-price behavior [11]. Stability and stochastic price dynamics were studied by Davies et al. [12], Osu et al. [13], and Adeosun et al. [14]. Ofomata et al. developed a stochastic stock-price forecasting model [15]; Osu [16] and Ugbebor et al. [17] also examined capital-market price changes. Standard stochastic-calculus arguments underlying these models can be applied alongside related work on random-parameter populations and stochastic differential equations [19–21].



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Existing studies motivate stochastic modeling, but a simple decay-adjusted GBM can make a specific stress scenario easier to interpret. The objective of this paper is therefore to derive the model, establish existence and uniqueness, simulate representative paths, and characterize sensitivity to the decay parameter. Importantly, the proposed process is a GBM with a reduced constant drift; it is not mean-reverting in the strict sense because it contains no state-dependent pull toward a fixed equilibrium level.

2. Materials and Methods

2.1. Standard geometric Brownian motion

Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$ be a filtered probability space satisfying the usual conditions, and let $\{W_t\}_{t\geq 0}$ be a standard Brownian motion. The standard GBM satisfies

$$dS_t = \mu S_t \, dt + \sigma S_t \, dW_t, \qquad S_0 > 0, \tag{1}$$

where $\mu \in \mathbb{R}$ is the drift and $\sigma \geq 0$ is the volatility.

Theorem 2.1 (Itô's formula)**.** *If X_t satisfies $dX_t = a(t, X_t)\, dt + b(t, X_t)\, dW_t$ and $f \in C^{1,2}$, then*

$$df(t, X_t) = \left(f_t + a f_x + \frac{1}{2} b^2 f_{xx} \right) dt + b f_x \, dW_t. \tag{2}$$

Applying Itô's formula to $f(S) = \ln S$ in Equation (1) gives

$$d \ln S_t = \left(\mu - \frac{1}{2}\sigma^2 \right) dt + \sigma \, dW_t, \tag{3}$$

and hence

$$S_t = S_0 \exp\left[\left(\mu - \frac{1}{2}\sigma^2 \right) t + \sigma W_t \right]. \tag{4}$$

2.2. Exponential-decay GBM

To represent persistent stress, introduce a decay parameter $k > 0$ and define

$$dS_t = (\mu - k) S_t \, dt + \sigma S_t \, dW_t, \qquad S_0 > 0. \tag{5}$$

The deterministic baseline associated with the decay component alone is $S_{\text{base}}(t) = S_0 e^{-kt}$. The full model combines this decay with ordinary GBM growth and volatility.

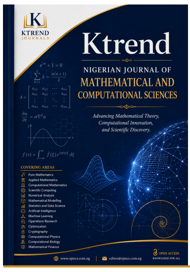
The formulation uses the following assumptions:

1. $S_0 > 0$;
2. $\mu \in \mathbb{R}$ and $k > 0$ are finite constants;
3. $\sigma \geq 0$ is a finite constant; and
4. W_t is a standard Brownian motion adapted to $\{\mathcal{F}_t\}$.

Theorem 2.2 (Existence, uniqueness, and explicit solution)**.** *Under the stated assumptions, Equation (5) has a unique strong solution,*

$$S_t = S_0 \exp\left[\left(\mu - k - \frac{1}{2}\sigma^2 \right) t + \sigma W_t \right]. \tag{6}$$

Moreover, $S_t > 0$ almost surely for every finite $t \geq 0$.



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Proof. The drift $b(s) = (\mu - k)s$ and diffusion coefficient $a(s) = \sigma s$ are globally Lipschitz and satisfy a linear-growth condition; therefore, a unique strong solution exists. For the explicit form, apply Itô's formula to $Y_t = \ln S_t$:

$$dY_t = \left(\mu - k - \frac{1}{2}\sigma^2\right) dt + \sigma\, dW_t. \tag{7}$$

Integrating from 0 to t and exponentiating yields Equation (6). Positivity follows immediately from the exponential representation. □

An equivalent derivation uses $Z_t = e^{kt} S_t$. The product rule gives

$$dZ_t = \mu Z_t\, dt + \sigma Z_t\, dW_t, \tag{8}$$

so Z_t is a standard GBM and $S_t = e^{-kt} Z_t$, which again produces Equation (6).

2.3. Distributional properties and stress metrics

From Equation (6),

$$\ln S_t \sim \mathcal{N}\left(\ln S_0 + \left(\mu - k - \frac{\sigma^2}{2}\right) t,\, \sigma^2 t\right). \tag{9}$$

Consequently,

$$\mathbb{E}[S_t] = S_0 e^{(\mu-k)t}, \tag{10}$$

$$\mathrm{Var}(S_t) = S_0^2 e^{2(\mu-k)t} \left(e^{\sigma^2 t} - 1\right). \tag{11}$$

When $k > \mu$, the expected price declines exponentially and its half-life is

$$t_{1/2} = \frac{\ln 2}{k - \mu}. \tag{12}$$

This quantity provides a direct, interpretable stress metric. When $k = \mu$, the expected price remains at S_0; when $k < \mu$, it grows at net rate $\mu - k$.

2.4. Simulation procedure

For a step size Δt, exact discrete simulation of Equation (5) is

$$S_{t+\Delta t} = S_t \exp\left[\left(\mu - k - \frac{\sigma^2}{2}\right)\Delta t + \sigma\sqrt{\Delta t}\, Z_t\right], \tag{13}$$

where the Z_t are independent standard normal random variables. Exact lognormal stepping avoids the negative simulated prices that can occur under a coarse Euler–Maruyama discretization.

3. Results and Discussion

3.1. Brownian-motion paths

Figure 1 presents three sample paths of the driving Brownian motion. All paths start at the same point, but their subsequent trajectories diverge. This illustrates how identical initial conditions can produce materially different outcomes once random increments enter the model. These paths provide the stochastic input in Equation (6).

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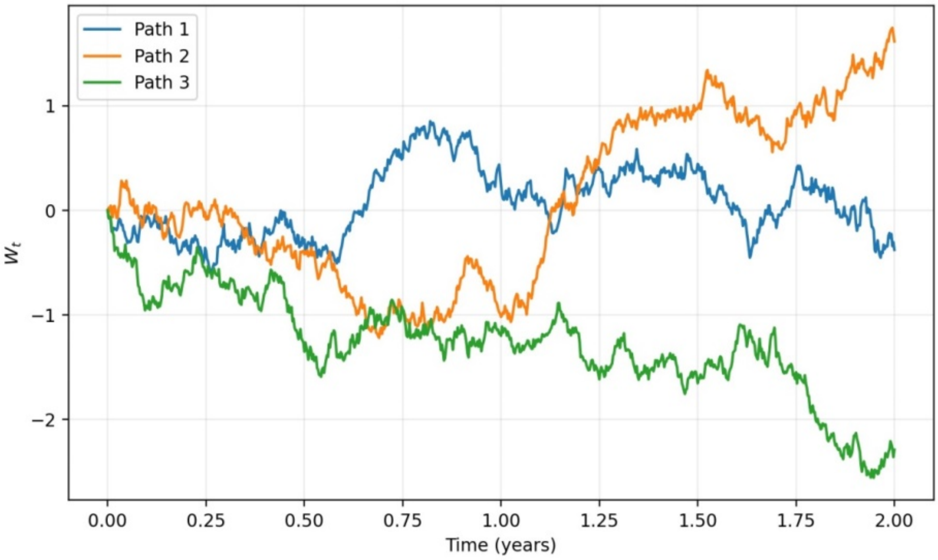

Figure 1: Three sample Brownian-motion paths used to illustrate the stochastic driver.

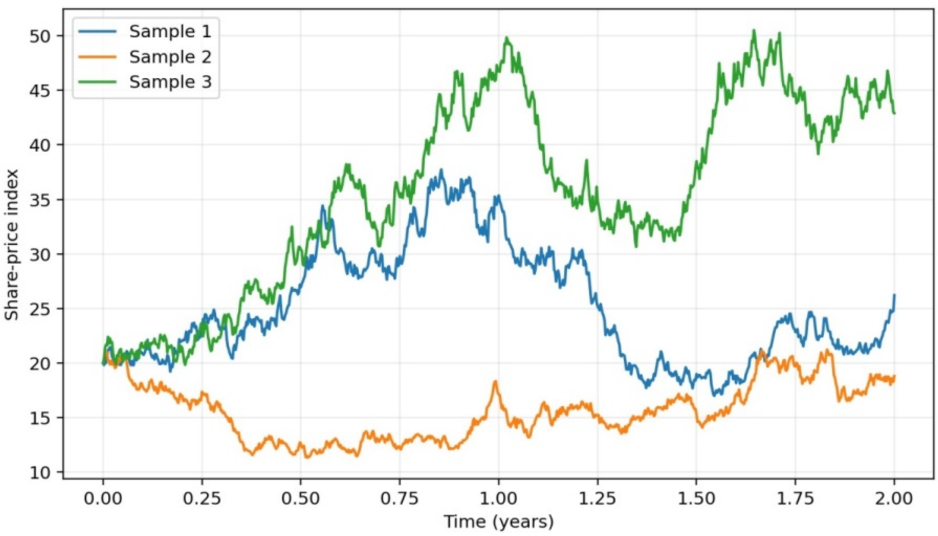

Figure 2: Illustrative sample paths from the exponential-decay GBM.

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3.2. Decay-adjusted stock-price paths

Figure 2 displays illustrative price paths from the decay-adjusted SDE. The paths can move together initially and then separate because each realization receives different Brownian increments. A positive decay parameter lowers the drift of every path, but it does not eliminate short-run upward movements generated by volatility. Thus, a declining expected value should not be confused with monotone decline in each simulated realization.

3.3. Sensitivity to the decay parameter

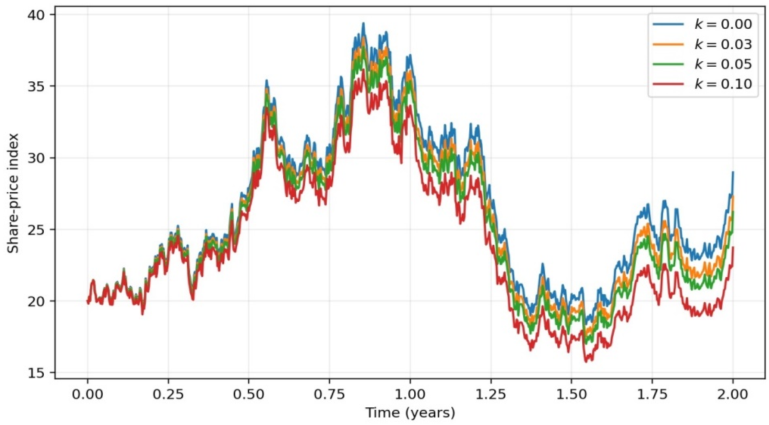

Figure 3: Sensitivity of the expected price to the decay parameter k.

Equation (10) establishes three regimes. If $k > \mu$, then $\mathbb{E}[S_t]$ decreases exponentially, representing sustained stress. If $k = \mu$, the expected value remains constant. If $k < \mu$, the expected value rises, representing growth that exceeds decay pressure. Figure 3 illustrates this dependence. The parameter k therefore controls the strength of the decay adjustment, while σ controls dispersion rather than the mean.

For risk management, the model can be used to compare scenarios by varying k, μ, and σ. However, the figures should be interpreted as demonstrations of model behavior. No bank-level data, parameter-estimation procedure, goodness-of-fit test, or out-of-sample validation is supplied in the present study. Accordingly, the results do not establish that the model accurately describes Access Bank, First Bank, or any other specific institution.

4. Conclusion

The exponential-decay GBM provides a transparent framework for generating bank-stock stress scenarios. Its explicit solution is positive and unique, and the parameter relation $k \gtrless \mu$ determines whether the expected price declines, remains stable, or grows. When $k > \mu$, the half-life $\ln(2)/(k - \mu)$ summarizes the speed of expected deterioration. Volatility still permits individual paths to rise temporarily, even when the mean declines.

The model should be described as a decay-adjusted GBM, not as a strictly mean-reverting process. A genuine mean-reverting price model would require state-dependent drift toward an equilibrium level, as in an Ornstein–Uhlenbeck-type or other mean-reverting specification. Future work should estimate parameters from Nigerian bank-stock data, define regulatory shock scenarios using observable covariates, compare the model with mean-reverting alternatives, and evaluate predictive and stress-testing performance out of sample.

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Conflict of Interest

The authors declare no conflict of interest.

Data Availability

No external dataset was analyzed in this theoretical and illustrative study. The mathematical derivations and simulation specification required to reproduce the model are provided in the article.

Author Contributions

All authors contributed to the conception of the study, development and interpretation of the mathematical model, manuscript preparation, and critical revision. All authors read and approved the final manuscript.

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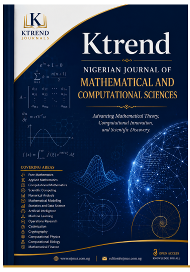

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